PHYS 798C Fall 2025
Lecture 27 Summary

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I. QUANTUM COMPUTING

Much has been written and discussed about the advantages of quantum computing over classical computing for certain classes of computational problems, such as quantum simulation. The advantages of quantum computing include i) the use of the superposition principle to put a quantum system in a superposition of two or more states, and ii) entanglement, in which the wavefunction describing multiple quantum systems are intricately inter-woven in a manner unique to quantum mechanics. This latter feature is the key to increasing the power of a quantum computer, but is difficult to achieve in practice for substantial periods of time.

The most obvious object to use to store and manipulate quantum information is the atom, since it was the first object to show clear quantum properties. However, superconductors offer the opportunity to build larger structures that are easier to fabricate in large scale integration that also show quantum properties, despite the fact that they are made up of many atoms. The unique phase-coherent ground state of a BCS superconductor enables it to act as a kind of macroscopic atom.

II. SUPERCONDUCTING QUBITS

Superconducting quantum bits (qubits) had their origins in demonstration of macroscopic quantum tunneling (MQT) in the 1980's. These were the first experiments to take objects that showed macroscopic quantum effects, such as flux quantization and the Josephson effect, and then take them to a *microscopic* quantum level. These experiments demonstrated that macroscopic superconducting devices could have discrete quantum energy levels, and that it was possible to manipulate these systems under unitary quantum evolution, at least to a limited extent. An overview of superconducting qubits is given on the class website, and discussed below.

In Lecture 23 we derived the tilted washboard potential $U(\gamma)$ for a current-biased Josephson junction, $U(\gamma) = \frac{-\hbar I_{dc}}{2e}\gamma - \frac{\hbar I_c}{2e}\cos\gamma$, where γ is the gauge-invariant phase difference of the Josephson junction. At finite current bias $I_{dc} < I_c$ there are a series of asymmetric potential wells for the "phase point" γ. The bottom of each of these wells has a certain curvature, characterized by the current-dependent plasma frequency $\omega_p(I_{dc})$ of the junction. The potential wells are finite on the large-γ side, and there is a finite probability that the phase point can tunnel through the barrier and turn into a running-state (finite voltage) state with $|d\gamma/dt| > 0$ (see slides 5 and 6 on the class website). The tunneling rate depends on the height and width of the barrier, and continuously evolves with increasing current bias. The curvature of the bottom of the well diminishes with increasing dc bias current as $\omega_p(I_{dc}) = \sqrt{\frac{2eI_c}{\hbar C}}\left(1 - \left(\frac{I_{dc}}{I_c}\right)^2\right)^{1/4}$. The discrete microscopic quantum energy states of the phase point are described by a harmonic oscillator eigenenergies as $E_n \approx (n + 1/2)\hbar\omega_p(I_{dc})$, with $n = 0, 1, 2, ...$, at least for the lowest states. This leads to a tunneling rate (as opposed to a thermally-activated escape rate) when the junction satisfies the condition that $\hbar\omega_p(I_{dc}) = 7.2k_BT$ (see slides 7 and 8 on the class website), which requires particular combinations of current bias and temperature.

The quantum levels are also evident in the tunneling rate measured as a function of microwave photon frequency. When a microwave photon of energy equal to the energy-level spacing between the ground state and an excited state is present, it can be absorbed by the junction, and the phase point moves to the higher energy state. In this state the barrier height and width of the tilted washboard are both smaller, leading to an enhanced tunneling rate. This kind of spectroscopy is shown in slide 9 on the class website.

The conditions required to achieve the microscopic quantum limit for the tilted washboard are to i) achieve the limit $k_BT \ll \hbar\omega_p$, and ii) to minimize the effects of loss by achieving $Q = \omega_pRC \gg 1$, using the R and C parameters from the RCSJ model of the Josephson junction. The device can then be used as a phase qubit.

An RF SQUID is a superconducting loop interrupted by a single Josephson junction. It has a potential energy as a function of gauge invariant phase difference that also has multiple minima, depending on the value of flux applied to the SQUID loop. When biased with flux near the half-quantum, $\Phi = \Phi_0/2$, and having a hysteretic design with $\beta_L = 2\pi L I_c/\Phi_0 > 1$, where L is the loop inductance, the RF SQUID can be put into a superposition of states in which the circulating current is flowing clockwise and counter-clockwise in the loop. This can be used as a so-called flux qubit. To achieve the quantum limit in this case requires that i) one operates at low temperature $k_B T \ll \hbar\omega_p$, where the relevant plasma frequency is the resonant frequency of the RF SQUID, $\omega_p = \frac{1}{\sqrt{\left(\frac{1}{L}+\frac{1}{L_{JJ}(T,\Phi_{app})}\right)^{-1} C}}$, and ii) the effects of loss are minimized by making $Q = \omega_p RC \gg 1$.

The Josephson inductance L_{JJ} is derived from the two Josephson equations. We can assign an inductance to the Josephson junction by calculating the voltage created due to a time-varying current applied to the junction: $V = L_{JJ}\frac{dI}{dt}$. Writing the voltage as $V = \frac{\Phi_0}{2\pi}\frac{d\gamma}{dt}$, and $\frac{dI}{dt} = I_c \cos\gamma \frac{d\gamma}{dt}$, we find that this reduces to $L_{JJ} = \frac{\Phi_0/2\pi}{I_c(T)\cos\gamma}$. Note that the critical current is temperature dependent, and the value of γ can be varied between 0 and 2π by means of flux Φ_{app} applied to the SQUID loop. Hence the Josephson inductance can take on both positive and negative values, and diverge, as demonstrated experimentally by Rifkin and Deaver.

More sophisticated superconducting qubits have been devised based on the above designs, along with other considerations. All of these qubits suffer from some degree of decoherence that arises from the fact that they are macroscopic objects that are subject to perturbations that range from infrared radiation and cosmic rays, to glassy two-level systems and non-equilibrium quasiparticles. In the next section we discuss one of these issues.

III. TWO-LEVEL SYSTEMS IN GLASSY/AMORPHOUS MATERIALS

Most superconducting qubits operate with transitions between the ground and excited state in the microwave frequency range, roughly 4-6 GHz, which corresponds to an energy of about 20 μeV. It has been known since the 1970's that a wide variety of solid materials can host atomic-scale defects that can tunnel between two local configurations. The energy scale for these tunneling events spans an enormous range of values due to the variety of defects that can form in materials. These so-called two-level systems (TLS) are a generic property of 'glassy' materials. They were discovered through measurements of heat capacity and thermal conductivity as a function of temperature of glassy and amorphous materials such as vitreous silica, germania, and selenium. The large heat capacity and its unusual temperature dependence has been explained in terms of a TLS with a wide range of energy scales (see e.g. W. A. Phillips, "Tunneling states in amorphous solids," J Low Temp Phys 7 (3), 351 (1972)).

Large numbers of these two-level systems can be embedded inside the structure of superconducting qubits. They can be found in surface oxides, or at other types of interfaces that are created in the deposition and patterning processes to create superconducting qubits. The population of TLS can be so high that one or more may share nearly the same transition energy as the macroscopic qubit. If there is interaction, either through an electric or magnetic dipole associated with the TLS and qubit, then the two can enter a new extended Hilbert space that often interferes with the operation of the quantum computer.

The TLS also impact the quality factor and resonance frequency of superconducting resonators that are coupled to qubits. The TLS have unusual absorptive properties for microwave photons. At low photon flux and low temperatures the TLS are mainly in their ground state and they have a large cross section for absorption, creating a substantial amount of loss. At higher photon flux and higher temperatures, the TLS find themselves mostly in the excited state and their absorption rate dramatically decreases. This behavior is illustrated with slides on the class website. A major issue in the development of superconducting quantum computing is the mitigation of TLS interactions with qubits and resonators.

A paper discussing microwave absorption and reactance of a bath of TLS is posted here.